Assignment #4:
Program Verification and Implementation

CSCI 5535 / ECEN 5533: Fundamentals of Programming Languages

Spring 2018: Due Friday, March 23, 2018

This homework has two parts. The first considers a deductive system for thinking about program correctness. The second considers a semantics that is closer a machine implementation.

1 Axiomatic Semantics: IMP

We continue to consider the same language IMP with the syntax chart:

Typ	τ	::=	`num`	`num`	numbers
			`bool`	`bool`	booleans
Exp	e	::=	`addr`[a]	a	addresses (or "assignables")
			`num`[n]	n	numeral
			`bool`[b]	b	boolean
			`plus`(e_1;e_2)	$e_1 + e_2$	addition
			`times`(e_1;e_2)	$e_1 * e_2$	multiplication
			`eq`(e_1;e_2)	$e_1 == e_2$	equal
			`le`(e_1;e_2)	$e_1 <= e_2$	less-than-or-equal
			`not`(e_1)	$!e_1$	negation
			`and`(e_1;e_2)	e_1 && e_2	conjunction
			`or`(e_1;e_2)	$e_1 \mid\mid e_2$	disjunction
Cmd	c	::=	`set`[a](e)	$a := e$	assignment
			`skip`	`skip`	skip
			`seq`(c_1;c_2)	c_1; c_2	sequencing
			`if`(e;c_1;c_2)	`if` e `then` c_1 `else` c_2	conditional
			`while`(e;c_1)	`while` e `do` c_1	looping
Addr	a				

As before, addresses a represent static memory store locations and are drawn from some unbounded set Addr and all memory locations only store numbers. A store σ is thus a mapping from addresses to numbers, written as follows:

$$\text{Store} \quad \sigma \quad ::= \quad \cdot \mid \sigma, a \hookrightarrow n$$

The semantics of IMP is as a formalized as before operationally, and we consider the Hoare rules for partial correctness as in Chapter 6 of *FSPL*.

1.1. **Program Correctness**. Prove using Hoare rules the following property: if we start the command `while` e `do` $a := a+2$ in a state that satisfies the assertion even(a), then it terminates in a state satisfying even(a). That is, prove the the following judgment:

$$\{\,\text{even}(a)\,\}\ \texttt{while}\ e\ \texttt{do}\ a := a+2\ \{\,\text{even}(a)\,\}$$

Hint: your proof should *not* use induction.

1.2. **Hoare Rules**. Consider an extension to IMP

$$c \quad ::= \quad \texttt{do}(c_1; e) \quad \texttt{do}\ c_1\ \texttt{while}\ e \quad \text{at-least-once looping}$$

with a command for at-least-once looping. Extend the Hoare judgment form $\{A\}\ c\ \{B\}$ for this command.

2 Abstract Machines and Control Flow

In this section, we will consider a new implementation of language PCF based on abstract machines (i.e., K from Chapter 28 of *PFPL*).

One aspect of a structural small-step operational semantics (as we used in previous assignments) that seems wasteful from an implementation perspective is that we "forget" where we are reducing at each step. An abstract machine semantics makes explicit the "program counter" in its state.

2.1. Give a specification for K as a call-by-value language. That is, modify the definition of the judgments f frame and $s \longmapsto s'$ from Section 28.1 of *PFPL*. You will also need to update the auxiliary frame-typing judgment $f : \tau \rightsquigarrow \tau'$ from Section 28.2 in order to state safety.

2.2. **Safety**.

(a) Prove preservation: if s ok and $s \longmapsto s'$, then s' ok.

(b) Prove progress: if s ok, then either s final or $s \longmapsto s'$ for some state s'.

2.3. **Implementation**.

(a) Implement call-by-value K. You need not include previously implemented language features (though you may include some of them if you want).

First, we have some new syntactic forms:

frames	f	**type** `frame`
stacks	k	**type** `stack = frame list`
states	s	**type** `state = Eval of stack * exp | Ret of stack * exp`

Then, we will implement functions that define both the static and dynamic semantics

of the language.

$[e'/x]e$	**val** `subst : exp -> var -> exp -> exp`
e val	**val** `is_val : exp -> bool`
$\Gamma \vdash e : \tau$	**val** `exp_typ : typctx -> exp -> typ option`
$s \longmapsto s'$	**val** `step : state -> state`
s final	**val** `is_final : state -> bool`
$k \triangleright : \tau$	**val** `stack_type : stack -> typ option`
$f : \tau \rightsquigarrow \tau'$	**val** `frame_type : frame -> typ -> typ option`
s ok	**val** `is_ok : state -> bool`
$s \hookrightarrow_{\mathsf{ok}} s'$	**val** `steps_pap : state -> state`

The $s \hookrightarrow_{\mathsf{ok}} s'$ is the analogous iterate-step-with-preservation-and-progress for states.

$$\frac{s\ \mathsf{ok} \qquad s\ \mathsf{final}}{s \hookrightarrow_{\mathsf{ok}} s} \qquad\qquad \frac{s\ \mathsf{ok} \qquad s \longmapsto s' \qquad s' \hookrightarrow_{\mathsf{ok}} s''}{s \hookrightarrow_{\mathsf{ok}} s''}$$

(b) **Extra credit: Exceptions**. Extend your K machine with exceptions as in Section 29.2. You may choose `nat` for the type of the value carried by the exception.

(c) **Extra credit: Continuations**. Extend your K machine with continuations as in Section 30.2. Implementing continuations is independent of implementing exceptions, so you may choose to do either or both. (Technically, you can encode exceptions with continuations.)

3 Final Project Preparation: Proposal Revision

3.1. **Reading Papers**. Follow some citations based on the papers you chose in Homework 2 and read in Homework 3. List at least three cited papers that seems relevant to follow up on. Include a citation along with a URL for each paper. For each of the additional papers, and for each question below, write two concise sentences:

(a) Why did *you* select this cited paper?

(b) What is the relation between the "main idea" of this cited paper and the "main idea" of the paper that cites it? You may want to skim the introductory and concluding bits of the cited paper along with the related work in the citing paper.

3.2. **Proposal**. Finalize your class project plan. Write an updated explanation of your plan (expanding and revising as necessary), and what you hope to accomplish with your project by the end of the semester. That is, on what artifact do you want to be graded?

Here are questions that you should address in your project proposal.

(a) Define the problem that you will solve as concretely as possible. Provide a scope of expected and potential results. Give a few example programs that exhibit the problem that you are trying to solve.

(b) What is the general approach that you intend to use to solve the problem?

(c) Why do you think that approach will solve the problem? What resources (papers, book chapters, etc.) do you plan to base your solution on? Is there one in particular that you plan to follow? What about your solution will be similar? What will be different?

(d) How do you plan to demonstrate your idea?

(e) How will you evaluate your idea? What will be the measurement for success?

4 Feedback and Discussion

4.1. **Assignment Feedback**. Complete the survey on the linked from the moodle after completing this assignment. Any non-empty answer will receive full credit for this part.

4.2. **Assignment Discussion**. Remember to sign up for a discussion session with your grader once you have received written feedback on your assignment. Engaging in a discussion session will receive full credit for this part.