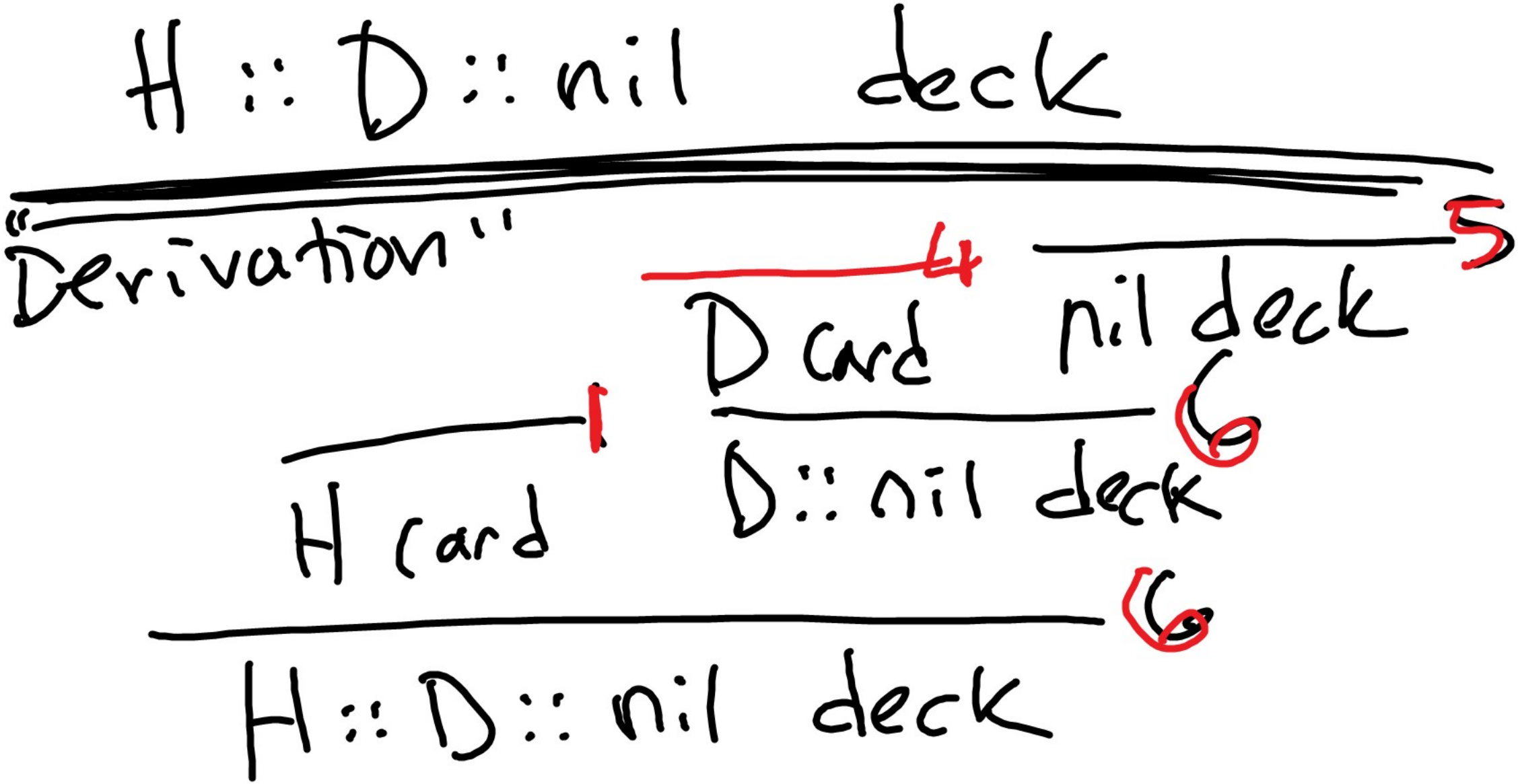
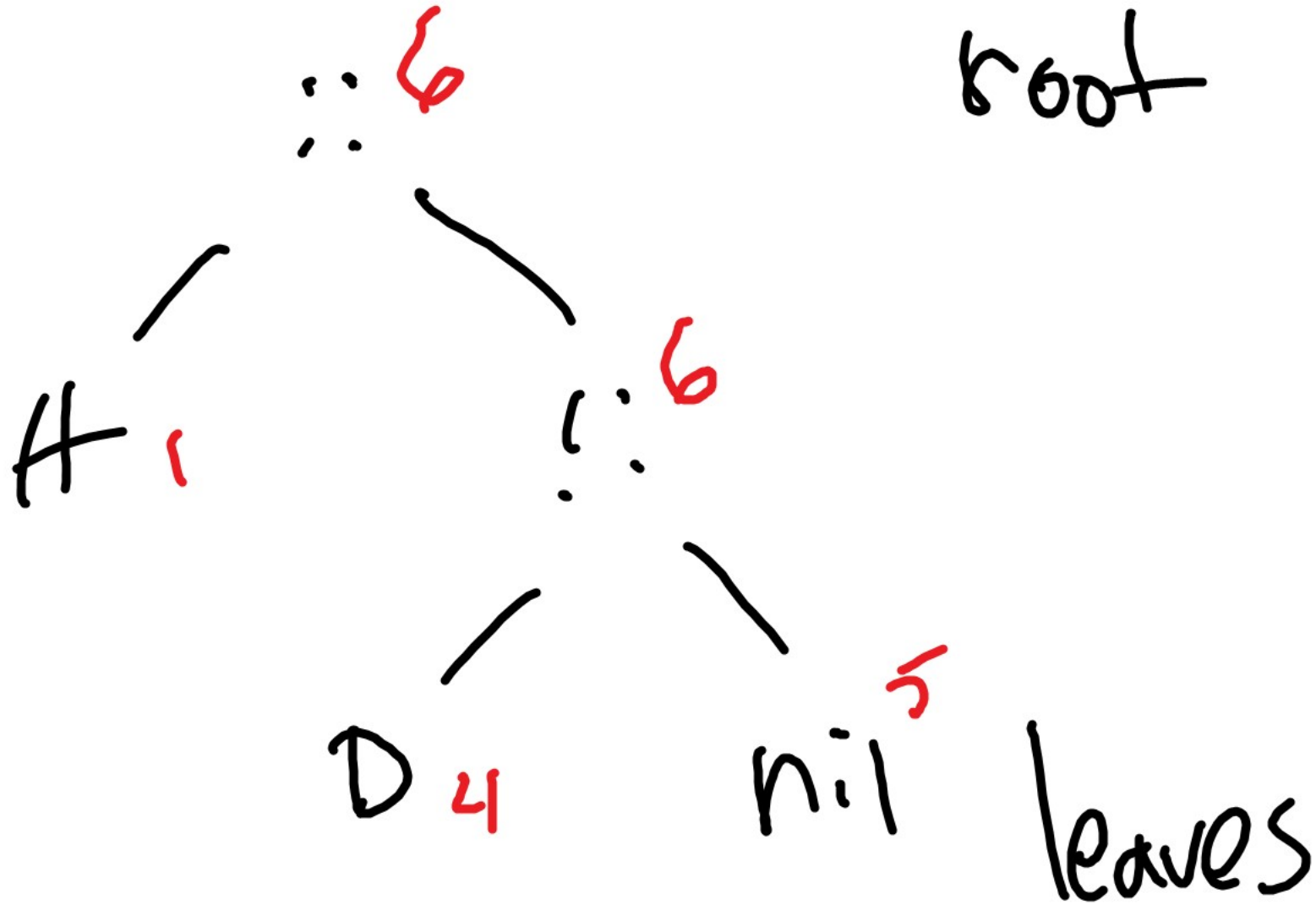
BNF - def syntax.

judgements — ang ind.
structure
(relations)

predicate — Unary relation

Cons(c, s) c::s


H :: D :: nil deck
"Derivation"
4
5
D card nil deck
1
6
H card
D :: nil deck
6
H :: D :: nil deck


::6
root
H1
::6
D4
nil5
leaves

$\forall s_1$ s_1 deck $\Rightarrow \exists s_2 s_3$ uns(s_1, s_2, s_3)

pf. by ind. over s_1 deck.

Case 5: $s_1 = nil$.

uns(nil, nil, nil) by rule 7.

Case 6: $s_1 = cons(c, s_1')$ $s_3' = s_3$

* uns(s_1', s_2', s_3') by IH.

uns$(c :: s_1', c :: s_2', s_3')$ by rule 9.

$$P(s_1') = \exists s_2' s_3' \; uns(s_1', s_2', s_3')$$

IF S deck $\Rightarrow uns(S, S', S'')$

THEN

```
1 :: 2 :: nil
```

```
[1, 2]
```

$$n ::= z \mid s(n)$$

n odd

$$\frac{n \text{ even}}{s(n) \text{ odd}}$$

n even

$$\frac{}{z \text{ even}} \qquad \frac{n \text{ odd}}{s(n) \text{ even}}$$

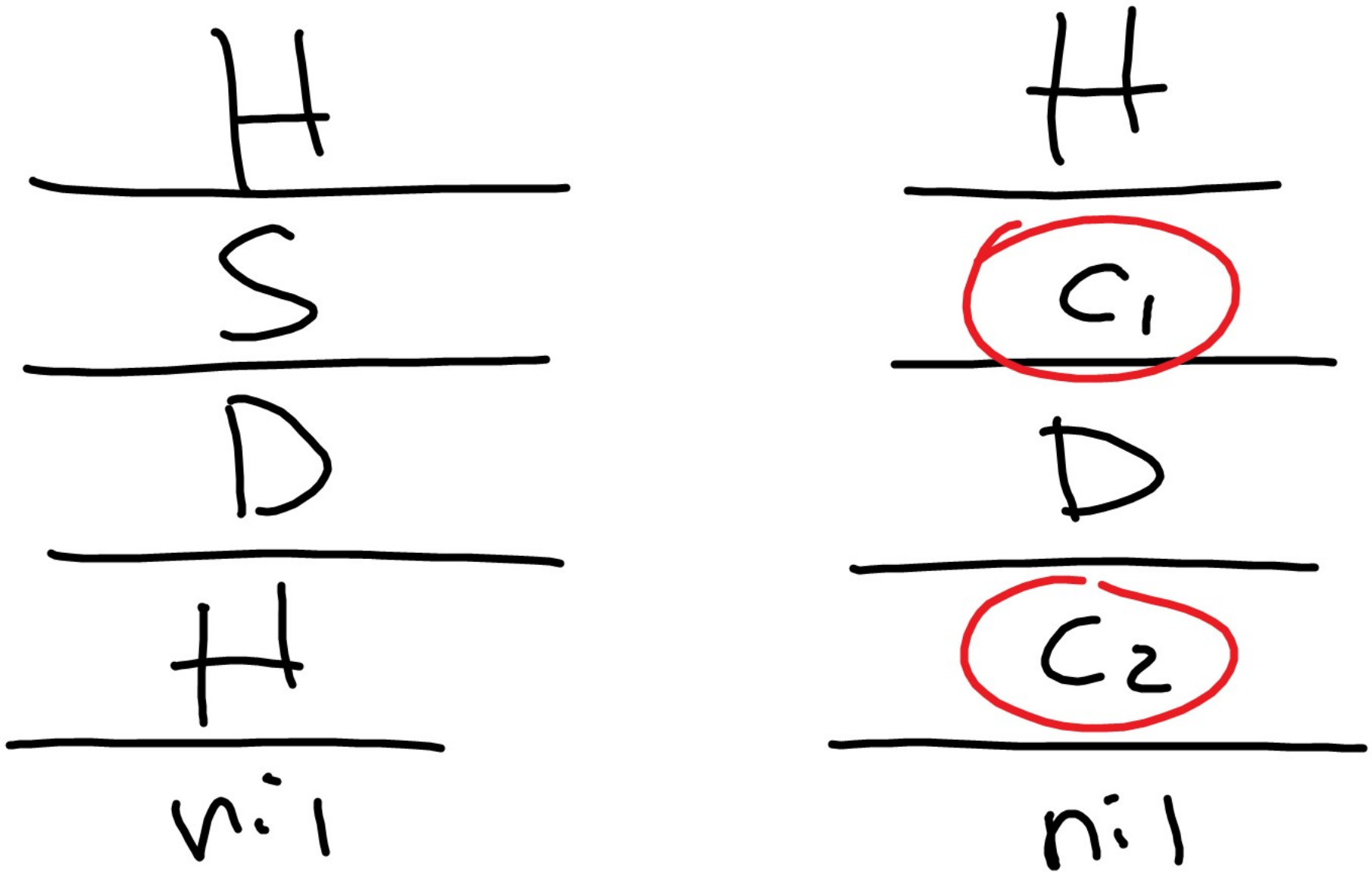
0	even
1	odd
2	even

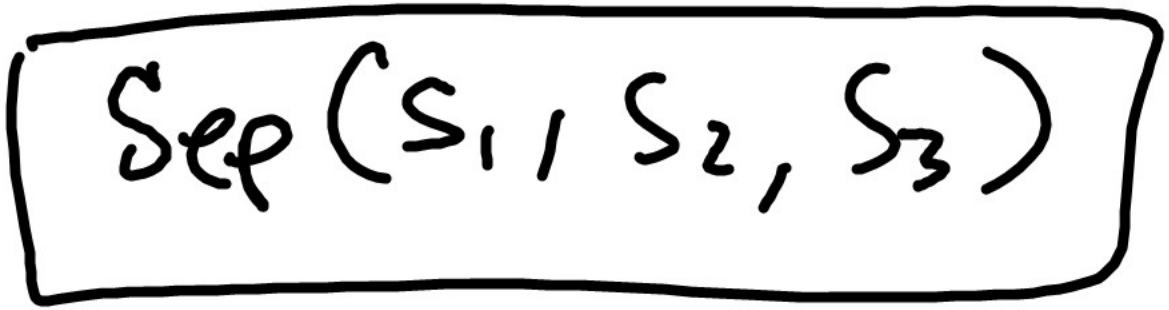
Hypothetical Judgements

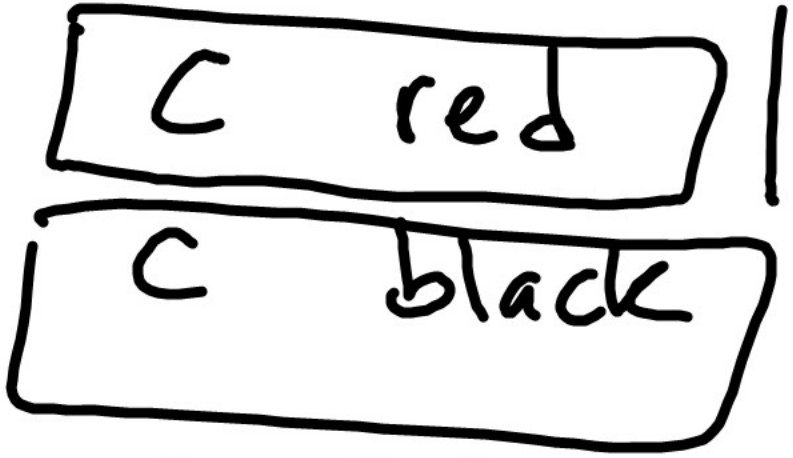
Typing judgement

$$\frac{\Gamma, x:\tau \vdash e : \tau'}{\Gamma \vdash \lambda x.e : \tau'}$$

"turnstile" $\vdash$


H
S
D
H
n:1
H
C1
D
C2
n:1





$$\frac{}{sep(nil, nil, nil)}\ 64$$

$$\frac{c\ \text{red} \qquad sep(S_1, S_2, S_3)}{sep(c :: S_1,\ c :: S_2,\ S_3)}\ 66$$

$$\frac{c\ \text{black} \qquad sep(S_1, S_2, S_3)}{sep(\underline{c} :: S_1, S_2, \underline{c} :: S_3)}\ 65$$

$\forall c.$

C black $\Rightarrow$ C Card

Pf. by ind over C black

Case $\frac{}{\S \text{ black}}\ 33$ — S card by $\frac{2}{\quad}$

Case $\frac{}{\clubsuit \text{ black}}\ 42$ — C card by $\frac{3}{\quad}$

$\forall s_1\ s_2\ s_3.$ $\quad sep(s_1, s_2, s_3)$

$$\Longrightarrow \quad s_2 \text{ dealt}$$
$$\wedge \quad s_3 \text{ dealt}$$

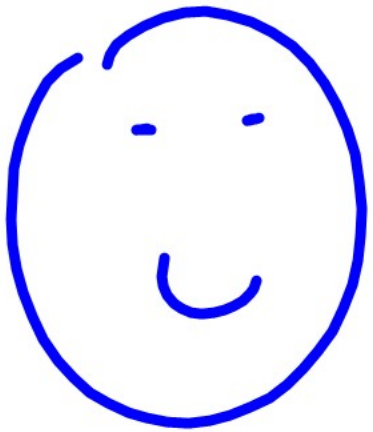
---

pf by ind over $sep(s_1, s_2, s_3)$

case 64 :

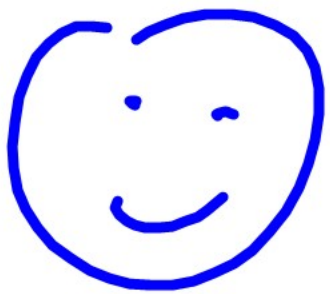
case $\frac{64}{65}$:

$\frac{}{66}$



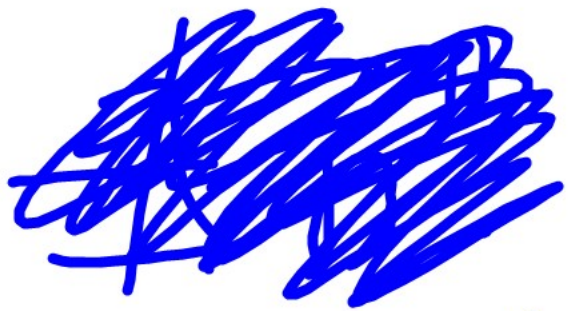
card

not derivable

b/c no rule for 

$$n ::= z \mid s(n)$$

↑ mutual exclusion.

$\exists$  $n' \neq s(n'')$

$\iff n' = z$

$$\forall s_1 \;.\; s_1\ \mathit{deck} \Rightarrow \exists s_2\ s_3\ \mathit{sep}(s_1, s_2, s_3)$$

---

pf by ind over s_1 deck.

Case $\dfrac{}{\underbrace{\mathit{nil}}_{s_1}\ \mathit{deck}}\,(5)$ $\quad \mathit{sep}(\mathit{nil}, \mathit{nil}, \mathit{nil})$ by 6 4

case $\dfrac{\mathit{Ccard}\ s_1'\ \mathit{deck}}{\underbrace{c :: s_1'}_{s_1}}\,(6)$ | $\mathit{sep}(s_1', s_2', s_3')$ by IH.
Case analysis on Ccard
...

Case $\dfrac{}{H \text{ cnd}}$

H red ---------- by rule ?

$\mathrm{Sep}(H :: S_1', H :: S_2', S_3')$ by rule 66

Case $\dfrac{}{S \text{ cnd}}$

S black ———————— by rule ?

$\mathrm{Sep}(S :: S_1', S_2', S :: S_3')$ by rule $6\bar{5}$

Lemma: $\forall c.\ c\ \mathrm{cmd} \Rightarrow (c\ \mathrm{red} \vee c\ \mathrm{black})$.

by ind over c cmd.

1 — by rule $(\mathrm{black})^{-1}$

2 — by rule $(\mathrm{red})^{-1}$

3 — by rule $(\mathrm{red})^{-1}$

4 — by rule $(\mathrm{black})^{-1}$

3.2