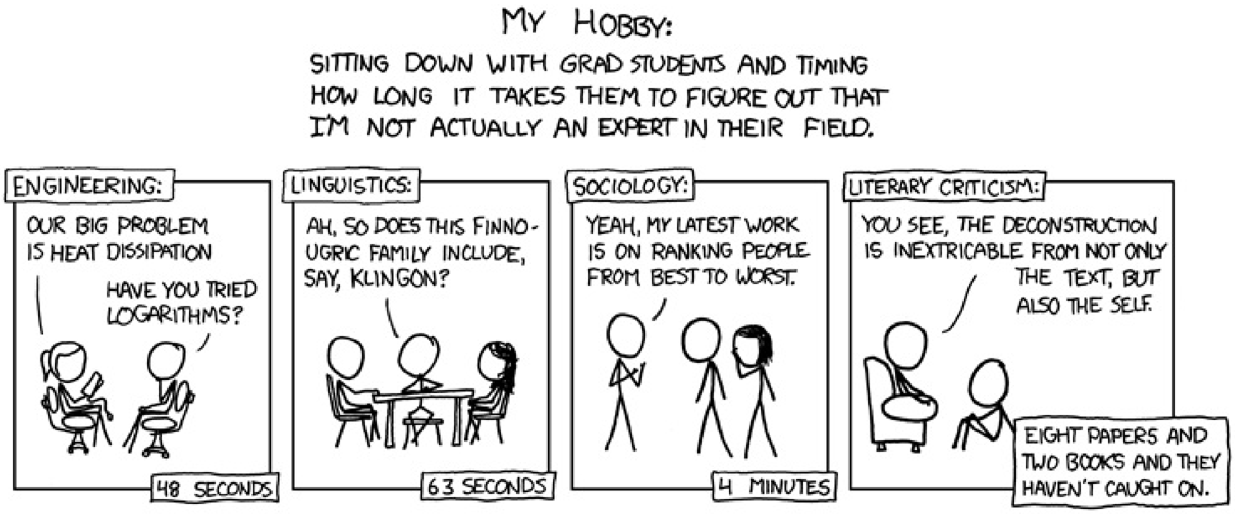
Meeting 7: Statics and Dynamics



Announcements

- Homework 1 due this week: Friday at 6:00pm

Questions

- Eirian
    - Work through Lemma 4.4 (Substitution)
        - Notation: dom; e1/e2/z
    - Lemma 4.1: Unicity of typing
        - Example of not satisfying
    - Chapter 3
        - Notation from 3.4
        - "Structural properties" -- big picture
- Sananda
    - Chapter 4
    1. How is rule 4.1 h formulated and what is alpha-equivalence?
    2. Did not quite understand the lemmas especially Lemma 4.2(Inversion for Typing), Lemma 4.4(Substitution) and Lemma 4.5(Decomposition).
    - Chapter 5
    1. What is By-value and By-name interpretation in Rule 5.4g
    2. Vague understanding of evaluation context under Contextual Dynamics (Holes and Ectxt) .
    3. Proof of Theorem 5.4.

iv. Notations of Equational Dynamics seem really complicating and confusing and hence could not follow the rules, or the equivalence and congruence relations.

Questions

1. Evaluation Contexts
2. Weakening + Substitution — Look at induction
3. Logistics — Can you use lemmas in the book?
   — Yes! Except where noted

Assignment #1:
Dynamics and Statics for a Simple Language

CSCI 5535 / ECEN 5533: Fundamentals of Programming Languages

Spring 2018: due Friday, February 9, 2018

The tasks in this homework ask you to prove ("meta theoretical") properties about the language **E**, defined in Part II of *PFPL*. They are "meta theoretical" in that they give a theory about the theory of **E**, and are true about a large set of **E** programs, not specific, individual programs. Language **E** is, by design, tiny to focus on the essence of meta-theory of programming languages.

Tasks

There is a lot of error checking going on in the dynamics in Appendix A. But as we will been discussing in class, we can eliminate much or all of this by equipping our language with a static type system (see Chapter 6 of PFPL)!

The type checking rules for **E** are reproduced in Appendix B for your reference.

Grading criteria: To receive full credit for any proof below, you must *at least* do the following:

- At the beginning of your proof, specify over what structure or derivation you are performing induction (i.e., which structure's inductive principle are you using?)
- In the inductive cases of the proof, specify how you are applying the inductive hypothesis, and what result it gives you.

If you omit these steps and/or do not make them explicit, you will receive zero credit for your proof. If you attempt to do these steps, but you make a mistake, you may still receive some partial credit, depending on your proof.

Note on omitting redundant proof cases: In the proofs below, some cases are very similar to other cases, e.g., the cases for `plus` and `times` in the proofs below are likely to be analogous, in that (nearly) the same proof steps are used in each. When this happens, you can omit the redundant cases as follows: If you do one case, say for `plus`, you may (optionally) write in the other case for `times` that it is "analogous to the case above, for `plus`". You must make this omission explicit, to show that you have thought about it. Further, this shortcut is only applicable when the cases really are analogous, and (nearly) the same steps apply in the proof. **When in doubt, do not omit the proof case.**

Task 1 (5 pts). (Canonical Forms) Prove that if e val, then

1. if $\Gamma \vdash e : \mathtt{num}$ then $e = \mathtt{num}[n]$ for some number n.

2. if $\Gamma \vdash e : \texttt{str}$ then $e = \texttt{str}[s]$ for some string s.

Task 2 (15 pts). (Substitution) State and prove the *substitution lemma* for **E**.

Task 3 (20 pts). (Progress) State and prove the *progress theorem* for **E**. To receive full credit, you must additionally use the canonical forms lemma correctly, when appropriate.

Task 4 (20 pts). (Preservation) State and prove the *preservation theorem* for **E**. To receive full credit, you must additionally use the substitution lemma correctly, when appropriate.

Task 5 (15 pts). (Language Extension and Run-Time Errors) Complete the formalization of the extension of **E** with $\texttt{div}(e_1;e_2)$ from Section 6.3 of PFPL. That is, extend the judgment forms $e\ \mathsf{val}$, $e \longmapsto e'$, $e\ \mathsf{err}$, $e \Downarrow e'$, and $\Gamma \vdash e : \tau$ from the appendix with rules for expression $\texttt{div}(e_1;e_2)$ as appropriate.

Task 6 (25 pts). (Relating Structural and Evaluation Dynamics) Prove that your structural dynamics for **E** with $\texttt{div}(e_1;e_2)$ coincide with your evaluation dynamics. Specifically, prove if $e \longmapsto^* e'$ and $e'\ \mathsf{val}$, then $e \Downarrow e'$. Hint: you will need a nesting of two inductions, that is, you will need to state and prove a separate lemma that you use in the proof of this main theorem.

A Dynamics of E

$\boxed{e\ \mathsf{val}}$

$$\frac{}{\texttt{num}[n]\ \mathsf{val}} \qquad \frac{}{\texttt{str}[s]\ \mathsf{val}}$$

$\boxed{e \longmapsto e'}$

$$\frac{}{\texttt{plus}(\texttt{num}[n_1];\texttt{num}[n_2]) \longmapsto \texttt{num}[n_1+n_2]} \qquad \frac{e_1 \longmapsto e_1'}{\texttt{plus}(e_1;e_2) \longmapsto \texttt{plus}(e_1';e_2)}$$

$$\frac{e_2 \longmapsto e_2'}{\texttt{plus}(\texttt{num}[n_1];e_2) \longmapsto \texttt{plus}(\texttt{num}[n_1];e_2')} \qquad \frac{}{\texttt{times}(\texttt{num}[n_1];\texttt{num}[n_2]) \longmapsto \texttt{num}[n_1*n_2]}$$

$$\frac{e_1 \longmapsto e_1'}{\texttt{times}(e_1;e_2) \longmapsto \texttt{times}(e_1';e_2)} \qquad \frac{e_2 \longmapsto e_2'}{\texttt{times}(\texttt{num}[n_1];e_2) \longmapsto \texttt{times}(\texttt{num}[n_1];e_2')}$$

$$\frac{}{\texttt{cat}(\texttt{str}[s_1];\texttt{str}[s_2]) \longmapsto \texttt{str}[s_1\hat{\ }s_2]} \qquad \frac{e_1 \longmapsto e_1'}{\texttt{cat}(e_1;e_2) \longmapsto \texttt{cat}(e_1';e_2)}$$

$$\frac{e_2 \longmapsto e_2'}{\texttt{cat}(\texttt{str}[s_1];e_2) \longmapsto \texttt{cat}(\texttt{str}[s_1];e_2')} \qquad \frac{}{\texttt{len}(\texttt{str}[s]) \longmapsto \texttt{num}[|s|]} \qquad \frac{e \longmapsto e'}{\texttt{len}(e) \longmapsto \texttt{len}(e')}$$

$$\frac{e_1\ \mathsf{val}}{\texttt{let}(e_1;x.e_2) \longmapsto [e_1/x]e_2} \qquad \frac{e_1 \longmapsto e_1'}{\texttt{let}(e_1;x.e_2) \longmapsto \texttt{let}(e_1';x.e_2)}$$

$\boxed{e\ \mathsf{err}}$

$$\frac{}{\texttt{plus}(\texttt{str}[s];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{plus}(\texttt{num}[n];\texttt{str}[s])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{plus}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{plus}(\texttt{num}[n];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{times}(\texttt{str}[s];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{times}(\texttt{num}[n];\texttt{str}[s])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{times}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{times}(\texttt{num}[n];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{cat}(\texttt{num}[n];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{cat}(\texttt{str}[s];\texttt{num}[n])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{cat}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{cat}(\texttt{str}[s];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{len}(\texttt{num}[n])\ \mathsf{err}} \qquad \frac{e\ \mathsf{err}}{\texttt{len}(e)\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{let}(e_1;x.e_2)\ \mathsf{err}}$$

$\boxed{e \Downarrow e'}$

$$\frac{}{\texttt{num}[n] \Downarrow \texttt{num}[n]} \qquad \frac{}{\texttt{str}[s] \Downarrow \texttt{str}[s]} \qquad \frac{e_1 \Downarrow \texttt{num}[n_1] \qquad e_2 \Downarrow \texttt{num}[n_2]}{\texttt{plus}(e_1;e_2) \Downarrow \texttt{num}[n_1+n_2]} \qquad \frac{e_1 \Downarrow \texttt{num}[n_1] \qquad e_2 \Downarrow \texttt{num}[n_2]}{\texttt{times}(e_1;e_2) \Downarrow \texttt{num}[n_1 \cdot n_2]}$$

$$\frac{e_1 \Downarrow \texttt{str}[s_1] \qquad e_2 \Downarrow \texttt{str}[s_2]}{\texttt{cat}(e_1;e_2) \Downarrow \texttt{str}[s_1 s_2]} \qquad \frac{e \Downarrow \texttt{str}[s] \qquad |s| = n}{\texttt{len}(e) \Downarrow \texttt{num}[n]} \qquad \frac{e_1 \Downarrow e_1' \qquad [e_1'/x]e_2 \Downarrow e_2'}{\texttt{let}(e_1;x.e_2) \Downarrow e_2'}$$

B Statics of E

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{}{\Gamma \vdash \texttt{num}[n] : \texttt{num}} \qquad \frac{}{\Gamma \vdash \texttt{str}[s] : \texttt{str}} \qquad \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{plus}(e_1;e_2) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{times}(e_1;e_2) : \texttt{num}} \qquad \frac{\Gamma \vdash e_1 : \texttt{str} \qquad \Gamma \vdash e_2 : \texttt{str}}{\Gamma \vdash \texttt{cat}(e_1;e_2) : \texttt{str}} \qquad \frac{\Gamma \vdash e : \texttt{str}}{\Gamma \vdash \texttt{len}(e) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{let}(e_1;x.e_2) : \tau_2}$$

For all $\Gamma, x, \tau, e', \tau', e, \tau$

If $\mathcal{D} :: \Gamma, x{:}\tau \vdash e' : \tau'$ and $\mathcal{E} :: \Gamma \vdash e : \tau$

Then $\Gamma \vdash [e/x]e' : \tau'$

Proof By induction on (the derivation of) $\mathcal{D}$ $\Gamma, x{:}\tau \vdash e' : \tau'$

Case

$$\mathcal{D} = \dfrac{}{\Gamma, x : \tau \vdash x : \tau}\ \text{4.1a}$$

case $e' = x$

Note that $e' = x$, $\tau' = \tau$. (1)

To show $\Gamma \vdash [e/x]e' : \tau'$

that is $\Gamma \vdash [e/x]x : \tau$ b/c of (1)

that is $\Gamma \vdash e : \tau$ by the defn of subst

We have $\Gamma \vdash e : \tau$ by assumption (i.e., deriv $\mathcal{E}$)

Case

$$\mathcal{D} = \dfrac{}{\Gamma, x:\tau \vdash y : \tau'}$$

Note that $e' = y$ and $\Gamma = \Gamma', y:\tau'$ for some Γ'

...

$\mathcal{D} = \Gamma, x:\tau \vdash e' : \tau'$

Case $\mathcal{D}_1 = \vdots$ $\qquad \mathcal{D}_2 = \vdots$

$$\mathcal{D} = \dfrac{\Gamma, x:\tau \vdash e_1' : \mathsf{num} \qquad \Gamma, x:\tau \vdash e_2' : \mathsf{num}}{\Gamma, x:\tau \vdash \underbrace{\mathsf{plus}(e_1'; e_2')}_{e'} : \underbrace{\mathsf{num}}_{\tau'}}\,(4.1d)$$

We have $\Gamma \vdash [e/x]e_1' : \mathsf{num}$ $\quad \mathcal{E}_1$

by the i.h. on $\mathcal{D}_1$ derivation of $\Gamma, x:\tau \vdash e_1' : \mathsf{num}$

with the derivation of $\mathcal{E}$ $\Gamma \vdash e : \tau$

We have

$\mathcal{E}_2$ $\quad \Gamma \vdash [e/x]e_2' : \mathrm{num}$

by the i.h. on $\mathcal{D}_2$ with $\mathcal{E}$

Now to show

$$\Gamma \vdash [e/x]\,\mathrm{plus}(e_1'; e_2') : \mathrm{num}$$

that is

$\mathcal{F}$ $\quad \Gamma \vdash \mathrm{plus}([e/x]e_1'; [e/x]e_2') : \mathrm{num}$

by the def of substitution

So construct $\mathcal{F}$

$$\mathcal{F} = \frac{\mathcal{E}_1\ \ \Gamma \vdash [e/x]e_1' : \mathrm{num} \qquad \mathcal{E}_2\ \ \Gamma \vdash [e/x]e_2' : \mathrm{num}}{\Gamma \vdash \mathrm{plus}([e/x]e_1'; [e/x]e_2') : \mathrm{num}}\ 4.1d$$